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Resonance of exchange amplitude of Compton effect in the circularly polarized laser field

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Abstract. We present, for general relativistic case, a theoretical study of resonance of exchange amplitude when a photon is scattered by an electron in the field of a circularly polarized wave. Resonances are related to a virtual intermediate particle that falls within the mass shell. We find conditions when resonances occur in exchange amplitude. We derive the expressions for the resonant amplitudes and the differential cross-sections when the invariant intensity parameter of the laser field is small ($\eta \ll 1$) and the interference of direct and exchange amplitudes is absent. It is demonstrated that the resonant cross-section of scattering may be several orders of magnitude higher than the cross-section of Compton effect in the absence of the external field.

PACS. 34.50.Rk Laser-modified scattering and reactions – 12.20.-m Quantum electrodynamics

1 Introduction

The theoretical study of the quantum processes of the first order in the fine structure constant in the presence of the field of a plane electromagnetic wave dates back to the 1960s and is connected with the creation of lasers. With production of ultrahigh-power femtosecond lasers experimental testing of this study becomes possible. The results of series of experiments at SLAC are found to be in agreement with the theoretical predictions [1,2]. Theoretical and experimental investigations of the first order process in the laser field go on to the present day (see, for example [3–5]).

The analysis of quantum-electrodynamics processes of the second order in the fine structure constant in the laser field is complicated by computational difficulties and a cumbersome form of results. Completely analytical calculations are possible only in the particular cases (see, for example [6]). Characteristic feature of these processes is the appearance of the resonances which are related to a virtual intermediate particle that falls within the mass shell (see the monographs [7], the review [8] and the articles [9–22]). Oleinic and Belousov first point out existing resonances in Compton process in the external electromagnetic wave [9–11] but their studies have mainly a qualitative character. In our work [12] we find cross-section for the resonant scattering of a photon by an electron through direct diagram (see Fig. 1a) in the framework of resonant approximation.

The purposes of the present work are clarification of a condition of resonance of exchange amplitude and calculation of a resonant cross-section for scattering through exchange diagram (see Fig. 1b).

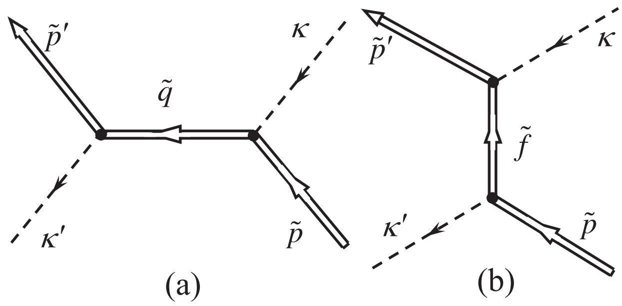

Fig. 1. Compton-effect in the field of a plane electromagnetic wave. The double lines correspond to the wave functions of an electron in the field of the wave (the Volkov functions), and the dashed lines represent a photon. (a) Direct diagram and (b) exchange diagram.

The relativistic system of units, where $\hbar = c = 1$, and standard metric $(ab) = a_0b_0 - \boldsymbol{ab}$ will be used throughout this paper.

2 Amplitude of scattering photon by electron in the laser field

Let us use the circularly polarized plane electromagnetic wave with the four-potential $A(\varphi)$ as a model of the laser field:

$$A(\varphi) = a(e_x \cos\varphi + \delta e_y \sin\varphi), \quad (1)$$

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where $a = F/\omega$; F and ω are the amplitude of the electric field strength and the frequency of the wave; $\delta = \pm 1$; $\varphi = (kx) = \omega t - \boldsymbol{k}\boldsymbol{x}$ is the phase; $k = (\omega, \boldsymbol{k})$ and $e_x = (0, \boldsymbol{e}_x)$, $e_y = \left(0, \boldsymbol{e}_y\right)$ are the four-momentum and the polarization four-vectors of the wave meeting the standard condition: $(e_x e_y) = 0$, $(e_x k) = (e_y k) = 0$, $e_x^2 = e_y^2 = -1$.

The amplitude of the scattering photon by an electron through exchange diagram is given by the expression (see Fig. 1b):

$$S = -ie^2 \int \mathrm{d}x \mathrm{d}x' \overline{\Psi}_{p'}(x) \gamma^\mu G(x, x') \gamma^\nu \Psi_p(x') \times A_\nu^*(\kappa' x') A_\mu(\kappa x), \quad (2)$$

where $p = (\varepsilon, \boldsymbol{p})$, $\kappa = (\kappa_0, \boldsymbol{\kappa})$ are four-momentums for an initial electron and photon; $p' = (\varepsilon', \boldsymbol{p}')$ and $\kappa' = (\kappa'_0, \boldsymbol{\kappa}')$ are four-momentums for a final electron and photon; γ^μ ($\mu = 0, 1, 2, 3$) is Dirac matrix; $A_\mu(\kappa x) = \sqrt{2\pi / \kappa_0}\, e_\mu \times \exp(-i(\kappa x))$ is the wave function of the photon; e_μ is the polarization four-vector of the photon; $\Psi_p(x)$ and $G(x, x')$ are the wave function and the Green-function (propagation function) electron in the field (1) [23–25]:

$$\Psi_p(x) = B_p(x) e^{iS_p(x)} \frac{u_p}{\sqrt{2\tilde{\varepsilon}}}, \qquad B_p(x) = 1 + \frac{e}{(kp)} \hat{k} \hat{A}. \quad (3)$$

$$G(x, x') = \frac{1}{(2\pi)^4} \int \mathrm{d}^4 q B_q(x) \frac{\hat{q} + m}{q^2 - m^2} \bar{B}_q(x') \times \exp\left(iS_q(x) - iS_q(x')\right), \quad (4)$$

where e and m are the charge and the mass of an electron; quantities with caps represent scalar products of a four-vector by Dirac matrixes; u_p is Dirac bispinor; $S_p(x)$ is the classical action of an electron in the field (1):

$$S_p(x) = -(px) - \frac{e}{(kp)} \int\limits_0^{(kx)} \left[(pA(\varphi)) - \frac{e}{2} A^2(\varphi)\right] \mathrm{d}\varphi. \quad (5)$$

We restrict our consideration to the case when the intensity of the external wave meets the condition

$$\eta = \frac{e\sqrt{-A^2}}{m} = \frac{eF}{m\omega} \ll 1. \quad (6)$$

In this case the amplitude (2) can be written as:

$$S = D e'^{*\nu} e^\mu \sum_{l=-\infty}^{\infty} \bar{u}_{p'} T_{\nu\mu}^{(l)} u_p \delta^{(4)}(\tilde{p} + \kappa + l\mathrm{k} - \tilde{p}' - \kappa'), \quad (7)$$

where $D = -ie^2 \left(4(2\pi)^3 \sqrt{\kappa_0 \varepsilon \kappa'_0 \varepsilon'}\right)^{-1}$ is a normalization constant; four-momentum with the tilde represents four-quasimomentum [24,25]:

$$\tilde{p} = p + \frac{m^2 \eta^2}{2(kp)} k. \quad (8)$$

In the expression (7) we introduce the notations:

$$T_{\nu\mu}^{(l)} = \sum_{l'=-\infty}^{\infty} \eta^{|l'|} \eta^{|l'-l|} \times M_\mu^{(l')}(p', f) \frac{\hat{f} + m}{f^2 - m^2} M_\nu^{(l-l')}(f, p), \quad (9)$$

where

$$\tilde{f} = \tilde{p} - \kappa' + l'k = \tilde{p}' - \kappa + (l' - l)k \quad (10)$$

is the four-quasimomentum of the intermediate electron; $M_\mu^{(l')}$ is the invariant amplitude which is proportional to the zeroth power of the parameter η. Taking into account the condition (6) and using the resonant approximation we need items only with the values $l' = \pm 1$:

$$M_\mu^{(\pm 1)}(p_1, p_2) = \mp \frac{1}{2} y(p_1, p_2) e^{\pm \mathrm{i}\chi} \gamma_\mu + \frac{m}{2(kp_2)} \left(k^\mu \hat{e}^{(\mp)} - e^{(\mp)\mu} \hat{k}\right) + \frac{m}{4} \left(\frac{1}{(kp_1)} - \frac{1}{(kp_2)}\right) \hat{e}^{(\pm)} \hat{k} \gamma_\mu. \quad (11)$$

Here $e^{(\pm)} = e_x \pm i\delta e_y$ are the polarization vectors of photons of the electromagnetic wave; $y(p', q)$ and $\chi \equiv \chi(p', q)$ are the kinematic parameters, which are written as

$$y(p_1, p_2) = \sqrt{-g^2(p_1, p_2)}, \quad \tan\chi = \frac{\delta(ge_y)}{(ge_x)}, \quad (12)$$

where we use the notation: $g \equiv g(p_1, p_2) = p_2/(kp_2) - p_1/(kp_1)$.

The amplitude (7) is proportional to $\eta^{|l'|} \eta^{|l'-l|}$ therefore we may restrict our consideration by the processes with small significance of the integer numbers l and l'. Each power of the parameter η meets the process of interaction with one photon of the wave. Note that the term which is proportional to the zeroth power of the parameter η ($l = l' = 0$) in the expression (7) determines amplitude of the Compton-effect in the absent external field. Hence we can make a conclusion that a correction term to the cross-section of Compton-effect in the low intense electromagnetic wave is proportional to the second order parameter η and therefore is small. Still the situation changes when a virtual intermediate electron falls within the mass shell:

$$f^2 = m^2. \quad (13)$$

In the first approximation this condition may be satisfied for $l = 0$, $l' = 1$ through electronic and for $l = 0$, $l' = -1$ through positronic intermediate states. Corresponding four-momentums we denote as (in the first approximation we take $\tilde{p} = p$, $\tilde{p}' = p'$)

$$f_- = p - \kappa' + k = p' - \kappa + k, \quad (14)$$

$$f_+ = \kappa' + k - p = \kappa + k - p', \quad (15)$$

where minus and plus conform to the electronic and positronic intermediate states accordingly.

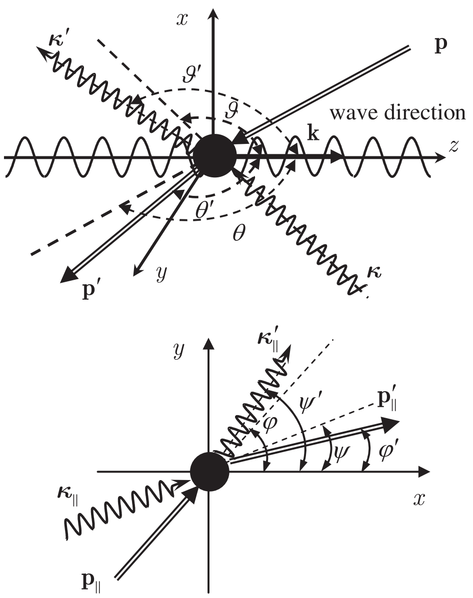

Fig. 2. The geometry for the study of Compton-effect in the field of a plane electromagnetic wave.

We have common four-momentum conservation law for both cases:

$$p + \kappa = p' + \kappa'. \tag{16}$$

As it follows from the equation (16) the frequency of the final photon is connected with the frequency of the initial photon by relationship

$$\kappa_0' = \frac{\kappa_0(1 - v\cos\tilde{\theta})}{1 - v\cos\tilde{\theta}' + (\kappa_0/\varepsilon)(1 - \cos\tilde{\vartheta}')}, \tag{17}$$

where $\boldsymbol{v} = \boldsymbol{p}/\varepsilon$ is the velocity of the initial electron; $\tilde{\theta} = \angle(\boldsymbol{p}, \boldsymbol{\kappa})$, $\tilde{\theta}' = \angle(\boldsymbol{p}, \boldsymbol{\kappa}')$, $\tilde{\vartheta}' = \angle(\boldsymbol{\kappa}, \boldsymbol{\kappa}')$ and $\cos\tilde{\theta}$, $\cos\tilde{\theta}'$, $\cos\tilde{\vartheta}'$ are given by

$$\cos\tilde{\theta} = \cos\theta\cos\vartheta + \sin\theta\sin\vartheta\cos(\varphi - \psi), \tag{18}$$

$$\cos\tilde{\theta}' = \cos\theta\cos\vartheta' + \sin\theta\sin\vartheta'\cos(\varphi - \psi'), \tag{19}$$

$$\cos\tilde{\vartheta}' = \cos\vartheta\cos\vartheta' + \sin\vartheta\sin\vartheta'\cos(\psi - \psi'). \tag{20}$$

Here

$$\theta = \angle(\boldsymbol{k}, \boldsymbol{p}), \qquad \vartheta = \angle(\boldsymbol{k}, \boldsymbol{\kappa}), \qquad \vartheta' = \angle(\boldsymbol{k}, \boldsymbol{\kappa}'), \tag{21}$$

$$\varphi = \angle(\boldsymbol{e}_1, \boldsymbol{p}_\parallel), \quad \psi = \angle(\boldsymbol{e}_1, \boldsymbol{\kappa}_\parallel), \quad \psi' = \angle(\boldsymbol{e}_1, \boldsymbol{\kappa}_\parallel') \tag{22}$$

are the polar (21) and azimuthal (22) angles of the initial electron, the initial and final photons correspondingly; $\boldsymbol{p}_\parallel$, $\boldsymbol{\kappa}_\parallel$, $\boldsymbol{\kappa}_\parallel'$ are components of the vectors $\boldsymbol{p}$, $\boldsymbol{\kappa}$, $\boldsymbol{\kappa}'$ which are parallel to the polarization plane (see Fig. 2).

Depending on the scattered angle of the final photon with respect to the direction of the momentum of the initial photon and electron, the frequency of the scattered photon falls within the interval

$$\frac{\kappa_0(1 - v\cos\theta)}{1 + (\kappa_0/\varepsilon) + |\boldsymbol{j}_1|} \le \kappa_0' \le \frac{\kappa_0(1 - v\cos\theta)}{1 + (\kappa_0/\varepsilon) - |\boldsymbol{j}_1|}, \tag{23}$$

where $\boldsymbol{j}_1 = \boldsymbol{v} + (\kappa_0/\varepsilon)\boldsymbol{n}_\kappa$.

Rigorously, the divergence of the amplitude of scattering in the resonance range indicates that expansion into a perturbation series is inapplicable in the situation under study. Correct calculation of the amplitude of scattering requires an approach that would fall beyond the framework of the perturbation theory. Specifically, we can perform summation of a principal sequence of Feynman diagrams. In practice, such summation is reduced to a consideration of radiative corrections to the masses of particles involved in the process under investigation. This procedure leads to a finite width of a resonance [7–22].

We use a resonant approaching to obtain a resonant amplitude and cross-section. In accordance with it everywhere with the exception of the denominator $f_\mp^2 = m^2$ and in the dominator the mass of an electron in the wave field becomes complex:

$$m \to m_* = m - i\Gamma/2, \tag{24}$$

where the width Γ is determined by the total probability of decay of intermediate state, i.e., the probability of single emission [24, 25]:

$$\Gamma = \frac{(f_\mp)_0}{m} W_1 = \frac{e^2 m}{4} \eta^2 F\left(u_1^{(\mp)}\right), \tag{25}$$

where minus and plus correspond to a resonance through electronic and positronic intermediate states and we introduce the notation

$$F\left(u_1^{(\mp)}\right) = \left(1 - \frac{4}{u_1^{(\mp)}} - \frac{8}{\left(u_1^{(\mp)}\right)^2}\right) \ln\left(1 + u_1^{(\mp)}\right) + \frac{1}{2} + \frac{8}{u_1^{(\mp)}} - \frac{1}{2\left(1 + u_1^{(\mp)}\right)^2}, \tag{26}$$

where $u_1^{(\mp)}$ are invariant parameters:

$$u_1^{(\mp)} = \frac{2(p\kappa')}{m^2} = \frac{u_1}{\tilde{u}' \pm 1}, \qquad \tilde{u}' = \frac{(\mathrm{k}\kappa')}{(p\kappa')}. \tag{27}$$

Behavior of the width Γ (25) on the parameters $u_1^{(\mp)}$ is shown in Figure 3.

We note that in the vicinity of resonance the cross-section obtained on the basis of resonance approaching is defined by a formula of the Breit-Wigner type [7–22].

Thus under resonant conditions $f_\mp^2 = m^2$ the amplitude is written in the following form :

$$S_{\text{res}}^{(\mp)} = \frac{D}{f_\mp^2 - m^2} e'^{*\nu} e^\mu \delta^{(4)}(p + \kappa - p' - \kappa') \times \bar{u}_{p'} \left[M_\mu^{(\pm 1)}(p', f_\mp)(\hat{f}_\mp + m) M_\nu^{(\mp 1)}(f_\mp, p)\right] u_p. \tag{28}$$

3 Kinematics of resonance

3.1 Electronic intermediate state

In the case of the resonance through electronic intermediate state we can rewrite a process as a consequence of two

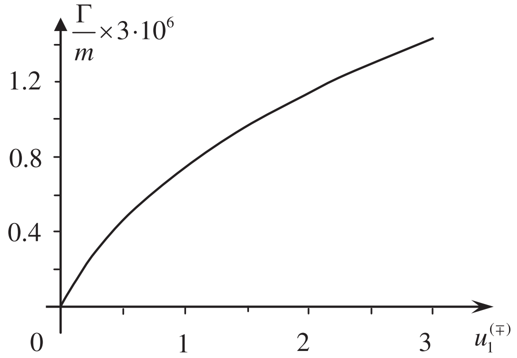

Fig. 3. Dependence the width resonance Γ on invariant parameter $u_1^{(\mp)}$ for $\eta = 0.05$.

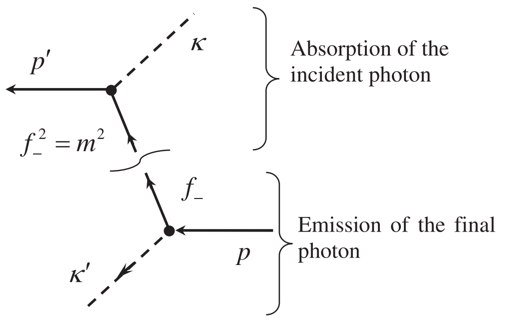

Fig. 4. Resonance in the exchange diagram through electronic intermediate state.

subprocesses (see Fig. 4): an emission of the final photon by the initial electron in the field of the wave and an absorption of the incident photon by an intermediate electron in the field of the wave. The conservation laws of four-momentum which correspond to these subprocesses have the following forms:

$$p + k = f_- + \kappa', \tag{29}$$

$$f_- + \kappa = p' + k. \tag{30}$$

Taking into account equation (14) the resonance conditions $f_-^2 = m^2$ can be rewritten in the laboratory frame of reference as

$$\kappa'_{0,\mathrm{res}} = \frac{\omega(1 - v\cos\theta)}{1 - v\cos\tilde{\theta}' + (\omega/\varepsilon)(1 - \cos\vartheta')}. \tag{31}$$

On account of positiveness of frequency (31) it must hold inequality:

$$u_1 = \frac{2(kp)}{m^2} = \frac{2\omega\varepsilon(1 - v\cos\vartheta)}{m^2} \le 1 \Rightarrow \tag{32}$$

$$\omega \le \frac{m^2}{2\varepsilon(1 - v\cos\theta)}, \tag{33}$$

where u_1 is an invariant parameter which determines the relativity of the process of scattering an electron by the wave [25] (the condition $u_1 \ll 1$ corresponds to non relativism and $u_1 \ge 1$ corresponds to relativism).

Equating expressions (17) and (31) we derive an expression that allows us to determine an interval of the frequency of the incident photon and the direction of propagation of a final photon for given initial electron energies and fixed parameters of the wave. We can conclude that the resonance through electronic intermediate state occurs in the exchange amplitude when the frequency of the initial photons lays in the interval:

$$\frac{\omega f}{1 + \sqrt{u_1\tau}} \le \kappa_0 \le \frac{\omega f}{1 - \sqrt{u_1\tau}}, \tag{34}$$

where

$$f = f(\theta, \tilde{\theta}) = \frac{(1 - v\cos\theta)}{(1 - v\cos\tilde{\theta})} \tag{35}$$

and τ is a kinematic invariant parameter:

$$\tau = \frac{(k\kappa)}{(p\kappa)} = \frac{\omega(1 - \cos\vartheta)}{\varepsilon(1 - v\cos\tilde{\theta})}, \tag{36}$$

which lays in the interval: $0 \le \tau \le u_1$.

For small frequency of the wave ($\omega \ll m$) the interval of the resonant frequencies of the incident photon also is small: $\Delta\kappa_{0,\mathrm{res}} \approx 4f^{3/2}\omega(\omega/m)\sin\vartheta/2$. Nevertheless we assume that this interval is greater than a spectral width of the laser: $\Delta\kappa_{0,\mathrm{res}} \gg \Gamma_\omega$ for $\vartheta \sim 1$.

The dependence of the resonant polar angle ϑ' of the final photon on the azimuthal angles φ' is given by

$$\vartheta'_{\mathrm{res}} = 2\arctan\left(\frac{\cos\theta_h\cos(\psi_h - \psi') \pm \sqrt{D}}{\cos\alpha + \cos\theta_h}\right). \tag{37}$$

Here

$$D = \sin^2\theta_h\cos^2(\psi_h - \psi') + \cos^2\theta_h - \cos^2\alpha, \tag{38}$$

$$\cos\alpha = \frac{h_0}{|\boldsymbol{h}|}, \quad \cos\theta_h = \frac{(\boldsymbol{hn})}{|\boldsymbol{h}|}, \quad \tan\psi_h = \frac{(\boldsymbol{he}_y)}{(\boldsymbol{he}_x)}, \tag{39}$$

$$h = (h_0, \boldsymbol{h}) = \left(\frac{(kp)}{(\kappa p)} - 1\right)p + \left(\frac{(kp)}{(\kappa p)}\kappa - k\right), \tag{40}$$

$$h_0 = \left(\frac{f\omega}{\kappa_{0,\mathrm{res}}} - 1\right)\varepsilon + \omega(f - 1), \tag{41}$$

$$\boldsymbol{h} = \left(\frac{f\omega}{\kappa_{0,\mathrm{res}}} - 1\right)\boldsymbol{p} + \omega(f\boldsymbol{n}_\kappa - \boldsymbol{n}), \tag{42}$$

where θ_h, ψ_h are the polar and azimuthal angles of the vector $\boldsymbol{h}$ (42).

From all values of $\vartheta'_{\mathrm{res}}$ in the equation (37) we must take only the positive one (see Fig. 5). The formula (37) has simple geometric interpretation: a unit vector $\boldsymbol{n}'_\kappa$ is one of the rulings of the cone, which is made by the vector $\boldsymbol{h}$ (42) with the cone angle α (39) (see Fig. 6).

3.2 Positronic intermediate state

In the case of the resonance through positronic intermediate state we can rewrite a process as a consequence of

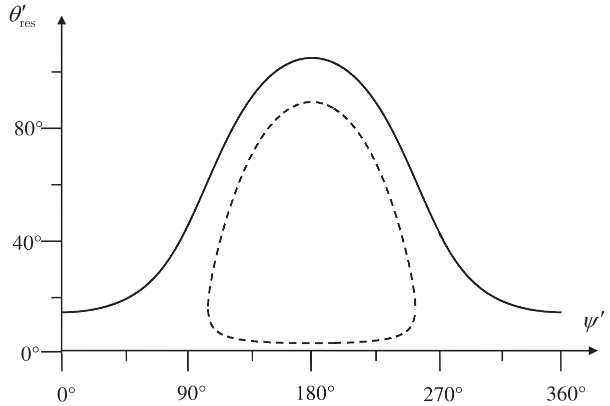

Fig. 5. Dependence of the polar angles θ' on the azimuthal angles of the final photon ψ' (Ex. (37)) for $\theta_h = 45°$ and $\psi_h = 180°$. Solid line corresponds $\alpha = 46°$, and dashed line corresponds $\alpha = 43°$.

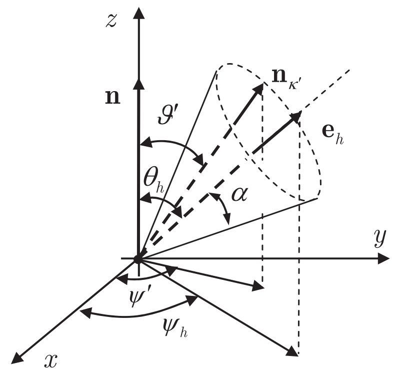

Fig. 6. Geometric interpretation of the formula (37).

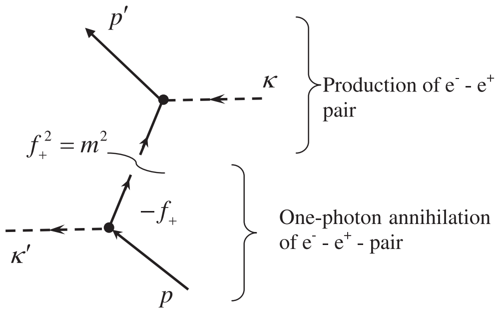

Fig. 7. Resonance in the exchange diagram through electronic intermediate state.

two subprocesses (see Fig. 7): production of an electron-positron pair by the indent photon in the field of the wave with consequent annihilation of electron-positron pair in the field of the wave. The conservation laws of four-momentum which correspond to these subprocesses have the following forms:

$$\kappa + k = f_+ + p', \tag{43}$$

$$f_+ + p = k + \kappa'. \tag{44}$$

Now, let us analyze the conditions when resonances appear through positronic intermediate state. Taking into account equation (15), the resonant conditions $f_+^2 = m^2$ can be rewritten in the laboratory frame of reference as

$$\kappa'_{0,\rm res} = \frac{\omega(1 - v\cos\theta)}{(\omega/\varepsilon)(1 - \cos\vartheta') - (1 - v\cos\tilde{\theta}')}. \tag{45}$$

On account of positiveness of the frequency (45) it must hold inequality:

$$u_1 \geq 1 \quad \Rightarrow \quad \omega \geq \frac{m^2}{2\varepsilon(1 - v\cos\theta)}. \tag{46}$$

Equating expressions (17) and (45) we may conclude that in the exchange amplitude the resonance through positronic intermediate state has a place when a frequency of the incident photon exceeds a certain threshold:

$$\kappa_0 \geq \kappa_{0,\rm lim}, \quad \kappa_{0,\rm lim} = \frac{\omega f}{\sqrt{u_1\tau} - 1} \tag{47}$$

where the kinematic invariant parameter τ (36) lays in the interval: $u_1^{-1} \leq \tau \leq u_1$.

The dependence of the resonant polar angle ϑ' of the final photon on the azimuthal angles φ' is given by formulas (37–39) with the replacements

$$h_0 \to \tilde{h}_0 = \left(\frac{f\omega}{\kappa_{0,\rm res}} + 1\right)\varepsilon + \omega(f - 1), \tag{48}$$

$$\boldsymbol{h} \to \tilde{\boldsymbol{h}} = \left(\frac{f\omega}{\kappa_{0,\rm res}} + 1\right)\boldsymbol{p} + \omega(f\boldsymbol{n}_\kappa - \boldsymbol{n}). \tag{49}$$

3.3 Interference of the resonant amplitudes

We can consider resonances of the direct and exchange (with electronic intermediate state) amplitudes separately with the exception of a case when a frequency of the incident photon lays not only in the interval (34) but it satisfies the expression:

$$\kappa_{0,\rm res} = \frac{\omega(1 - v\cos\theta)}{1 - v\cos\tilde{\theta} - (\omega/\varepsilon)(1 - \cos\vartheta)}. \tag{50}$$

This is a condition of the resonance of the direct amplitude [12]. Substituting the equation (50) in the expressions (41)–(42) we can find the resonant polar angles ϑ' (37) of the final photon which correspond to the interference of the direct and exchange amplitudes.

Interference of the resonant direct and exchange (with positronic intermediate state) amplitudes has no place because the maximal value of the frequency (50):

$$\kappa_{0,\rm res} = \frac{\omega(1 - v\cos\theta)}{1 - \omega/\varepsilon - |\boldsymbol{v} - (\omega/\varepsilon)\boldsymbol{n}|} \tag{51}$$

is less than a threshold value of a frequency of the incident photon (47) which is necessary for a passing of the resonance through positronic intermediate state.

4 Resonant cross-sections

The resonant cross-section for the direct amplitude averaged over polarizations of the initial photon and electron and summed over polarizations of the final photon and electron has been obtained in the work [12]. Similarly we obtain the resonant cross-section for the exchange diagram.

For a resonance of the exchange amplitude we have following expressions for a different resonant cross-sections averaged over polarizations of the initial photon and electron and summed over polarizations of the final photon and electron:

$$\frac{\mathrm{d}\sigma_{\mathrm{res}}^{(\mp)}}{\mathrm{d}\Omega'} = \frac{r_e^2\eta^4}{2}\left(\frac{\kappa_0'}{\Gamma}\right)^2 \frac{\tilde{u}_1^{-2}H^{(\mp)}(\tilde{u}',u_1,\tilde{u},\tilde{u}_1)}{1+R(u'_{\mathrm{res}}-u')^2}, \tag{52}$$

where $r_e = e^2/m$ is the classical radius of an electron; minus and plus correspond to the electronic and positronic states accordingly; u' is an invariant parameter and u'_{res} is a value of a parameter u' which corresponds to the resonance:

$$u' = \frac{(\kappa\kappa')}{(p\kappa')}, \qquad u'_{\mathrm{res}} = (\tilde{u}'\pm 1)\frac{\tilde{u}_1}{u_1} - 1, \tag{53}$$

a function R is given by:

$$R = \frac{m}{\Gamma}\frac{u_1^2}{(1+u')^2}. \tag{54}$$

In formula (52) the functions $H^{(\mp)}$ is written as

$$\begin{aligned} H^{(\mp)} &= f(\upsilon_2^{(\mp)},u_1^{(\mp)})f(\upsilon_1^{(\mp)},u_1^{(\mp)}) \\ &+ \frac{2\upsilon_1^{(\mp)}\upsilon_2^{(\mp)}}{(\upsilon_1^{(\mp)}\pm 1)(\upsilon_2^{(\mp)}\pm 1)}\left(\frac{\upsilon_1^{(\mp)}+\upsilon_2^{(\mp)}}{u_1^{(\mp)}} - \frac{2\upsilon_1^{(\mp)}\upsilon_2^{(\mp)}}{\left(u_1^{(\mp)}\right)^2}\right) \\ &+ g(\upsilon_2^{(\mp)},u_1^{(\mp)})g(\upsilon_1^{(\mp)},u_1^{(\mp)}) - \frac{2\upsilon_1^{(\mp)}\upsilon_2^{(\mp)}}{\left(u_1^{(\mp)}\right)^2}\left(\tilde{u}_1 - u_1^{(\mp)}\right). \end{aligned} \tag{55}$$

Here

$$f\left(\upsilon_1^{(\mp)},u_1^{(\mp)}\right) = 2 + \frac{\left(\upsilon_1^{(\mp)}\right)^2}{\upsilon_1^{(\mp)}\pm 1} - 4\frac{\upsilon_1^{(\mp)}}{u_1^{(\mp)}}\left(1-\frac{\upsilon_1^{(\mp)}}{u_1^{(\mp)}}\right), \tag{56}$$

$$g\left(\upsilon_1^{(\mp)},u_1^{(\mp)}\right) = \frac{(2\pm\upsilon_1^{(\mp)})(u_1^{(\mp)}-2\upsilon_1^{(\mp)})\upsilon_1^{(\mp)}}{2u_1^{(\mp)}(\upsilon_1^{(\mp)}\pm 1)}, \tag{57}$$

$$\upsilon_1^{(\mp)} = \frac{(\mathrm{k}\kappa')}{(\mathrm{k}p)} = \frac{\tilde{u}'}{\tilde{u}'\pm 1}, \tag{58}$$

$$\upsilon_2^{(\mp)} = \frac{(\mathrm{k}\kappa)}{(\mathrm{k}p')} = \frac{\tilde{u}}{1+\tilde{u}-\upsilon_1^{(\mp)}}, \tag{59}$$

$$\tilde{u} = \frac{(\mathrm{k}\kappa)}{(\mathrm{k}p)}, \quad \tilde{u}_1 = \frac{2(\kappa p)}{m^2}. \tag{60}$$

It is convenient to compare the resonant differential cross-section (52)–(55) with a differential cross-section of Compton-effect in the absence of the external field [26]:

$$\frac{\mathrm{d}\sigma_{\mathrm{c}}}{\mathrm{d}\Omega'} = 2r_e^2\left(\frac{\kappa'_0}{m}\right)^2\frac{C\,(u',\tilde{u}_1)}{\tilde{u}_1^2}. \tag{61}$$

Here

$$C\,(u',\tilde{u}_1) = 2 + \frac{u'^2}{1+u'} - 4\frac{u'}{\tilde{u}_1}\left(1-\frac{u'}{\tilde{u}_1}\right). \tag{62}$$

The ratio of the resonant differential cross-section (52) to the conventional different cross-section in the absence of the external field (61) for

$$|u'-u'_{\mathrm{res}}| \lesssim \frac{u'_{\mathrm{res}}+1}{u_1}\left(\frac{\Gamma}{m}\right)^2 \tag{63}$$

has a shark peak which may be written as

$$\frac{\mathrm{d}\sigma_{\mathrm{res}}^{(\mp)}}{\mathrm{d}\sigma_{\mathrm{c}}} = \frac{4}{\alpha^2}\frac{H^{(\mp)}(\tilde{u}',u_1,\tilde{u},\tilde{u}_1)}{F^2(u^{(\mp)})C\,(u',\tilde{u}_1)}, \tag{64}$$

where $\alpha = e^2 = 1/137$ is the fine-structure constant.

For electronic intermediate state the invariant parameters $\tilde{u}_1$, τ, $\tilde{u}'$ lay in the intervals:

$$\frac{u_1}{1+\sqrt{u_1\tau}} \le \tilde{u}_1 \le \frac{u_1}{1-\sqrt{u_1\tau}}, \tag{65}$$

$$0 \le \tau \le u_1, \qquad 0 \le \tilde{u}' \le u_1. \tag{66}$$

For positronic intermediate state the parameter $\tilde{u}_1$ must exceed the threshold:

$$\tilde{u}_1 \ge \tilde{u}_{1,\mathrm{lim}}, \quad \tilde{u}_{1,\mathrm{lim}} = \frac{u_1}{\sqrt{u_1\tau}-1} \tag{67}$$

and parameters τ, $\tilde{u}'$ lay in the intervals:

$$u_1^{-1} \le \tau \le u_1, \qquad 1+u_1^{-1} \le \tilde{u}' \le u_1. \tag{68}$$

In general relativistic case, the resonant cross-section of Compton effect in the field of the low intense circularly polarized electromagnetic wave can't be factorized into production of probabilities (or cross-sections) of two subprocesses of the first order in the fine-structure constant. However it is possible in the non relativistic case

$$u_1 \ll 1 \tag{69}$$

(all other invariant parameters in the expression (55) are less or equal to u_1) for the resonance through electronic intermediate state. Under condition (69) we can write

$$\frac{\Gamma}{m} = \frac{e^2\eta^2}{3}u_1(1-u_1), \tag{70}$$

$$H^{(-)} = f(\tilde{u}',u_1)f(\tilde{u},u_1), \tag{71}$$

here

$$f(\tilde{u}',u_1) = 2 - 4\frac{\tilde{u}'}{u_1}\left(1-\frac{\tilde{u}'}{u_1}\right). \tag{72}$$

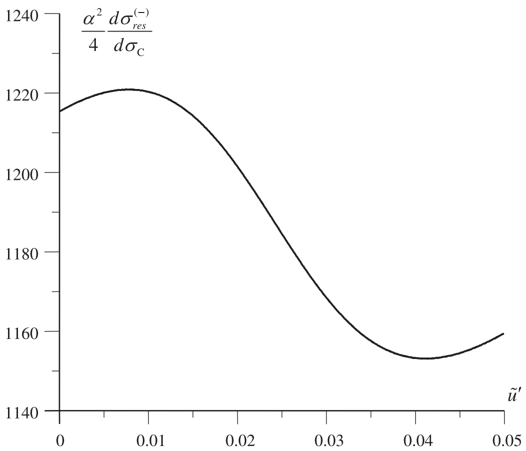

Fig. 8. The dependence of the ratio of the resonant differential cross-section (52) to the conventional different cross-section in the absence of the external field (61) on the invariant parameter $\tilde{u}'$ for $u_1 = 0.05$, $\tau = 0.5u_1$, $\tilde{u}_1 = 0.5$, $\tilde{u} = 0.015$.

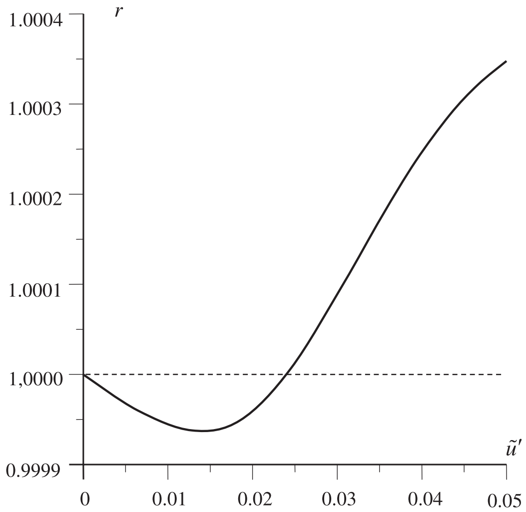

Fig. 9. The dependence of the factor of factorization r (75) on the invariant parameter $\tilde{u}'$ for $u_1 = 0.05$, $\tau = 0.5u_1$, $\tilde{u}_1 = 0.5$, $\tilde{u} = 0.015$.

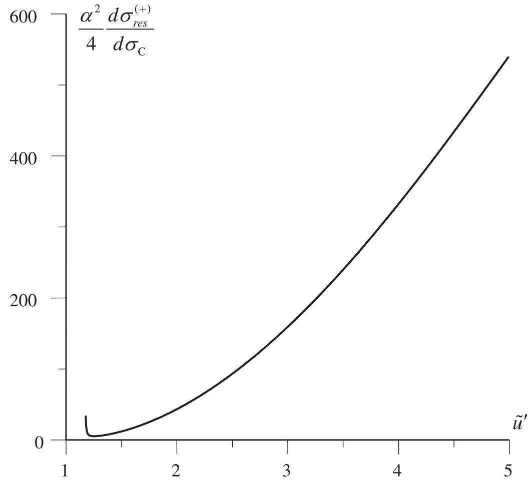

Fig. 10. The dependence of the ratio of the resonant differential cross-section (52) to the conventional different cross-section in the absence of the external field (61) on the invariant parameter $\tilde{u}'$ for $u_1 = 5$, $\tau = 0.05u_1$, $\tilde{u}_1 = 10$, $\tilde{u} = 5.99$.

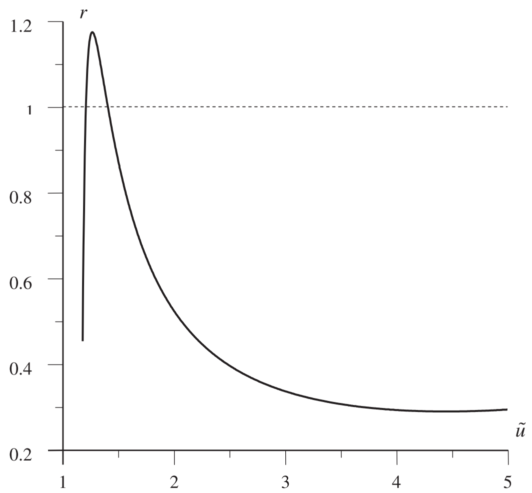

Fig. 11. The dependence of the factor of factorization r (75) on the invariant parameter $\tilde{u}'$ for $u_1 = 5$, $\tau = 0.5u_1$, $\tilde{u}_1 = 10$, $\tilde{u} = 5.99$.

In this case we can write the resonant cross-section in a factorized form:

$$\mathrm{d}\sigma_{\mathrm{res}}^{(-)} \sim \frac{\mathrm{d}w^e w^a}{1 + R(u'_{\mathrm{res}} - u')^2}, \tag{73}$$

where $\mathrm{d}w^e \sim f(\tilde{u}', u_1)\mathrm{d}\Omega'$ is the differential probability per unit time to emit the final photon with absorption of one photon of the electromagnetic wave, $w^a \sim f(\tilde{u}, u_1)$ is the probability per unit time to absorb the initial photon with radiation one photon of the electromagnetic wave.

Estimation shows that the resonant cross-section may be several orders of magnitude greater than the cross-section of the corresponding process in the absence of the external field

$$\frac{\mathrm{d}\sigma_{\mathrm{res}}^{(\mp)}}{\mathrm{d}\sigma_{\mathrm{c}}} \gtrsim \frac{4}{\alpha^2} \gg 1. \tag{74}$$

Figures 8–11 demonstrate dependence of the ratio of the resonant differential cross-section (52) to the conventional different cross-section in the absence of the external field (61) on the invariant parameter $\tilde{u}'$ and the dependence of the factor of factorization r (75):

$$r = \frac{f(\upsilon_2^{(\mp)}, u_1^{(\mp)}) f(\upsilon_1^{(\mp)}, u_1^{(\mp)})}{H(\tilde{u}', u_1, \tilde{u}, \tilde{u}_1)} \tag{75}$$

on the invariant parameter $\tilde{u}'$.

5 Conclusion

Analysis of Compton effect through the exchange diagram in the field circularly polarized electromagnetic wave has demonstrated that this process may occur in the resonant region.

The resonance in the exchange amplitude through electronic intermediate state has a place when it holds inequality (32) and the frequency of the incident photon lays in the interval (34).

The resonance in the exchange amplitude through positronic intermediate state has a place when it holds inequality (46) and the frequency of the incident photon exceeds a certain threshold (47).

Estimation shows that the resonant cross-section may be several orders of magnitude greater than the cross-section of the corresponding process in the absence of the external field:

$$\frac{\mathrm{d}\sigma_{\mathrm{res}}^{(\mp)}}{\mathrm{d}\sigma_{\mathrm{c}}} \gg 1.$$

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